Metal-insulator phase transition in an exactly solvable model of nondegenerate interacting fermion chains

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A two parametric family of the models of two coupled nondegenerate fermion chains is proposed and solved by the means of the Bethe ansatz. We give a detailed analysis of the exact zero-temperature phase diagram at half filling for different parameters of the interactions. It is shown that insulator and "interchain ferromagnetic" phase states are separated by "intermediate metallic" state, these phase states coexist in a tricritical point. The critical fluctuations are described by a conformal field theory with the central charge $c=1$. In the tricritical point the fermion state is described as a Luttinger liquid state with one gapless mode that is characterized by abnormal large density-density correlations and correlations of the momentum distribution function.

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I. INTRODUCTION

The metal-insulator transition in strongly correlated materials remains a central problem of modern condensed-matter physics. The quantum critical point separating different phases is a topic of great current interest.[1] The nondegenerate strongly interacting fermion systems have been subject of intensive investigation. Localized and itinerant electrons and their correlations in multiorbital systems seem to play an important role in the formation of heavy-fermion states in the transition-metal oxides and rare-earth compounds. One typical example of transition-metal oxide LiV_2O_4, in which heavy-fermionlike behavior was observed.[2] Mostly models of strongly interacting electron systems describing subbands with different bandwidths are used for description of such compounds, they permit to describe explicitly both the localized orbitals, such as the d in transition-metal oxides or the f in heavy-fermion systems, and their hybridization to an itinerant electron bands (p orbitals of oxygen in transition-metal oxides). The minimal model consists of a regular array of sites, each associated with nondegenerate f-electron orbital coupled to a delocalized conduction electron orbital. The Kondo lattice and periodic Anderson model should be specially noted.

The simpler case of spinless fermions, which shall be the subject of the investigation, can be realized in fully spin-polarized ultracold fermionic gases.[3] We shall show that strong correlations in itinerant bands yield a heavy mass and eventually induce a transition to the Mott insulating state.

II. HAMILTONIAN AND EXACT SOLUTION OF THE MODEL

In an effort to gain a better understanding of the behavior of such systems, we consider here the model of two free chains of nondegenerate spinless fermions in a zigzag fashion with the Hamiltonian $\mathcal{H}=\mathcal{H}_1+\mathcal{H}_2+\mathcal{H}_{int}-w_1N_1-w_2N_2$. The first and second terms take into account the hopping and correlated hopping fermions along each chains[4] but not from one chain to the other

$$\mathcal{H}_1=-\sum_{j=1/2}^{L-1/2}(c_j^\dagger c_{j+1}+c_{j+1}^\dagger c_j)[t_1+(t_2-t_1)n_{j+1/2}],$$

$$\mathcal{H}_2=-\sum_{j=1}^{L}(c_j^\dagger c_{j+1}+c_{j+1}^\dagger c_j)[t_2+(t_1-t_2)n_{j+1/2}], \quad (1)$$

the third term $\mathcal{H}_{int}$ takes into account the interaction between nearest-neighbor fermions of different chains

$$\mathcal{H}_{int}=J\sum_{j=1}^{L}[(n_j+n_{j+1})n_{j+1/2}-c_j^\dagger c_{j+1}c_{j+1+1/2}^\dagger c_{j+1/2}-c_{j+1}^\dagger c_j c_{j-1/2}^\dagger c_{j+1/2}], \quad (2)$$

where c_j and $c_j^\dagger$ are operators of spinless fermions at site j ($j=\frac{1}{2},\frac{3}{2},\frac{5}{2},L-\frac{1}{2}$ for the first and $j=1,2,\ldots,L$ for the second chains, L is assumed to be even), the particle number operator for fermions is defined by $n_j=c_j^\dagger c_j$, t_1 (t_2) and t_2 (t_1) are the hopping (correlated hopping) integrals of fermions of the first and second chains, respectively, J is the coupling constant. The difference of w_1 and w_2 defines the shift of the fermion subbands of the chains. The Hamiltonian conserves the total number of fermions in the chains N_1 and N_2.

Below we present the exact solution of the model obtained by the Bethe ansatz. The eigenvector $|\psi\rangle$ with $N=N_1+N_2$ particles is defined as $|\psi\rangle=\Sigma_{\{x_j,y_j\}}\psi(x_1,\ldots,y_1,\ldots)|x_1,\ldots,y_1,\ldots\rangle$, where the Bethe function takes a traditional form,

$$\psi(x_1,x_2,\ldots,x_{N_1},y_1,y_2,\ldots,y_{N_2})=\sum_{P_1,P_2}(-1)^{P_1+P_2}A(P_1,P_2)\exp\left(i\sum_{j=1}^{N_1}k_{P_1j}^{(1)}x_j+i\sum_{j=1}^{N_2}k_{P_2j}^{(2)}y_j\right), \quad (3)$$

where the P_1 and P_2 summations extend over all $N_1!$ and $N_2!$ the permutations P_1 of $\{1,\ldots,N_1\}$ the momenta $\{k_j^{(1)}\}$ of the particles of the first chain and the permutations P_2 of $\{1,\ldots,N_2\}$ the momenta $\{k_j^{(2)}\}$ of the particles of the second chain. The coefficients $A(P_1,P_2)$ are connected by the two-particle scattering S matrix that is the solution of the following Schrödinger equation for the two-particle Bethe function of spinless fermions on the nearest-neighbor lattice sites $x,x+\frac{1}{2}$,

$$E_2\psi(x,x+1/2) = -t_2\psi(x-1,x+1/2) - t_1\psi(x,x+3/2) - t_2\psi(x,x-1/2) - t_1\psi(x+1,x+1/2) + J\psi(x,x+1/2) - J\psi(x+1,x-1/2), \quad (4)$$

here $E_2=-2t_1\cos k^{(1)}-2t_2\cos k^{(2)}-w_1-w_2$, $k^{(1)}$ and $k^{(2)}$ are the momenta of fermions.

According to the form of the interaction in Eqs. (1) and (2) the fermions of one chain are scattered on the fermions of another only, the two-particle scattering matrix of the spinless fermions is the solution of the Schrödinger equation for the two-particle Bethe function (4),

$$S_{12} = \frac{2t_+\cos k_+ + 2it_-\sin k_+ + J\exp(-ik_-)}{2t_+\cos k_+ - 2it_-\sin k_+ + J\exp(ik_-)}, \quad (5)$$

here $k_\pm=\frac{1}{2}(k^{(1)}\pm k^{(2)})$ and $t_\pm=\frac{1}{2}(t_1\pm t_2)$.

An additional constrain on the form of the interaction in Eqs. (1) and (2) follows from the calculations of the configurations with three particles in different chains on the neighboring lattice sites. Let us consider the Schrödinger equation for the three-particle Bethe function of spinless fermions on the nearest-neighbor and next-nearest-neighbor lattice sites $x-1/2,x,x+1/2$,

$$\begin{aligned} E_3\psi(x-1/2,x,x+1/2) &= -t_1\psi(x-3/2,x,x+1/2) - t_1\psi(x-1/2,x,x+3/2) \\ &\quad - t_1\psi(x-1/2,x-1,x+1/2) \\ &\quad - t_1\psi(x-1/2,x+1,x+1/2) + 2J\psi(x-1/2,x,x+1/2), \end{aligned} \quad (6)$$

here $E_3=-2t_1\cos k^{(1)}-2t_2\cos k^{(2)}-2t_1\cos k^{(3)}-2w_1-w_2$.

The Bethe function describes noninteracting spinless fermions of the same chain $\psi(x\pm 1/2,x,x\pm 1/2)=0$, now making use of the fact that Eq. (6) is of the same structure as Eq. (4) in this case, we conclude that the following relation between amplitudes holds for arbitrary configuration of the particles for free fermion chains $A(k_{P'1},k_{P'2},k_{P'3}) = S(k_{Pj},k_{Pj+1})A(k_{P1},k_{P2},k_{P3})$ [here P is an arbitrary permutation and $P'=P(j,j+1)$].[5]

Below we present the exact solution of the model with open boundary conditions obtained by the Bethe ansatz. The boundary conditions lead to sets of the discrete Bethe equations for the momenta of fermions. The eigenstates of the model Hamiltonian are characterized by sets of the momenta $\{k_j^{(1)}\}(j=1,\ldots,N_1)$ and $\{k_j^{(2)}\}(j=1,\ldots,N_2)$ for the particles of each chain, that satisfy the following Bethe equations:

$$k_j^{(1)}L = 2\pi I_j^{(1)} + \sum_{p=1}^{N_2}\Phi(k_j^{(1)},k_p^{(2)},t_1,t_2),$$

$$k_j^{(2)}L = 2\pi I_j^{(2)} + \sum_{p=1}^{N_1}\Phi(k_j^{(2)},k_p^{(1)},t_2,t_1), \quad (7)$$

where $\Phi(k_j^{(1)},k_p^{(2)},t_1,t_2)=2\tan^{-1}(\frac{J\sin k_-+2t_-\sin k_+}{J\cos k_-+2t_+\cos k_+})$, $I_j^{(1)}$ and $I_j^{(2)}$ take positive integers. The energy of the system is given by $E=-2t_1\Sigma_{j=1}^{N_1}\cos k_j^{(1)}-2t_2\Sigma_{j=1}^{N_2}\cos k_j^{(2)}-w_1N_1-w_2N_2$.

III. GROUND-STATE PROPERTIES, RESULTS, AND DISCUSSION

We will briefly summarize the results of the exact solution of the ground state of the system in the thermodynamic limit. Let us redefine the hopping integrals as $t_1=1$ and $t_2=t$, the model is described by two parameters of the interaction t and J, where $0\le t\le 1$. The case $t>1$ is reduced to the exchange of the numbers of the chains. For $J>|1+t|$ the charge rapidities $\lambda_j^{(l)}$ $(j=1,\ldots,N_l)$ related to the momenta of particles

$$\exp(ik_j^{(l)}) = \pm\frac{\sin\left[\frac{1}{2}(\lambda_j^{(l)}+i\varphi_l)\right]}{\sin\left[\frac{1}{2}(\lambda_j^{(l)}-i\varphi_l)\right]}$$

are obtained by solving the Bethe ansatz equations,

$$\left[\frac{\sin\left\{\frac{1}{2}[\lambda_j^{(l)}+i\varphi_l]\right\}}{\sin\left\{\frac{1}{2}[\lambda_j^{(l)}-i\varphi_l]\right\}}\right]^L = \prod_{p=1}^{N_{l'}}\frac{\sin\left\{\frac{1}{2}[\lambda_j^{(l)}-\lambda_p^{(l')}+2i\varphi_+]\right\}}{\sin\left\{\frac{1}{2}[\lambda_j^{(l)}-\lambda_p^{(l')}-2i\varphi_+]\right\}}, \quad (8)$$

where $\coth\varphi_-=\sqrt{\frac{1-J^2/(2t_-)^2}{1-J^2/(2t_+)^2}}$, $\coth\varphi_+=-\frac{t_-}{t_+}\coth\varphi_-$, $\varphi_\pm=(\varphi_1\pm\varphi_2)/2$, here and below $l,l'=1,2$ denote the number of the chains and $l\ne l'$.

In the thermodynamic limit the solutions of Eq. (8) are described by the string hypothesis

$$\lambda_{jm_l}^{(l)} = \Lambda_{jm_l}^{(l)} + i(m_l-2k+1)\varphi_l, \quad k=1,\ldots,m_l, \quad (9)$$

where $\Lambda_{jm_l}^{(l)}$ are the centers of the m_l strings. These solutions define the complexes with m_1 and m_2 particles, one-particle states are described by the real rapidities $\lambda_{j1}^{(l)}$.

The fillings of the subbands are defined by the chemical potentials of the chains, which are the same for different subbands under their partial filling. This condition corresponds to the minimum of the ground-state energy under given total number of fermions. At strong splitting of the subbands, when their energies are not overlapped, the properties of the system are defined by a partially filled chain. In the ground state the real solutions k_j of the Bethe Eq. (8) exist at $J>J_c$, the complex solutions at $J<J_c$. The magnitude of $J_c=-1-t$ is the similar to the point $\Delta=-2$ in the XXZ spin-1/2 Heisenberg chain (HC), which separates antiferromagnetic gapless and ferromagnetic states. For $J<J_c$ the ground state of the chains is defined by the complexes with $2m$ particles $(m_1=m_2=m)$ at partial filling of the chain subbands, the total energy of this complex with zero total momentum is equal to $\varepsilon=\varepsilon_1+\varepsilon_2$, $\varepsilon_l=\frac{\sinh\varphi_l\sinh(m\varphi_l)}{\cosh(m\varphi_l)-1}$. These solutions correspond to $2m$-spinless fermions localized on the nearest-neighbor lattice sites. We call this insulator state as "interchain ferromagnetic" state, which is similar to a ferromagnetic state in HC.

Using an analogous parametrization for the momenta $\exp(ik_j^{(l)})=\frac{\sinh[\frac{1}{2}(\lambda_j^{(l)}+i\phi_l)]}{\sinh[\frac{1}{2}(\lambda_j^{(l)}-i\phi_l)]}$, we obtain the Bethe equations for other regions of the parameters $1-t\le J\le 1+t$ and $-1+t\le J\le -1-t$ in the form

$$\left[\frac{\sinh\left\{\frac{1}{2}[\lambda_j^{(l)}+i\phi_l]\right\}}{\sinh\left\{\frac{1}{2}[\lambda_j^{(l)}-i\phi_l]\right\}}\right]^L=\prod_{p=1}^{N_{l'}}\frac{\sinh\left\{\frac{1}{2}[\lambda_j^{(l)}-\lambda_p^{(l')}+2i\phi_+]\right\}}{\sinh\left\{\frac{1}{2}[\lambda_j^{(l)}-\lambda_p^{(l')}-2i\phi_+]\right\}},\tag{10}$$

where $\cot\phi_-=\sqrt{\frac{J^2/(2t_-)^2-1}{1-J^2/(2t_+)^2}}$, $\cot\phi_+=-\frac{t_-}{t_+}\cot\phi_-$, and $\phi_\pm=(\phi_1\pm\phi_2)/2$.

In the case $-1+t<J<1-t$ the Bethe equations are not reduced to the rapidity representation. In the thermodynamic limit the Bethe Eq. (7) (for real momenta k_j) are reduced to set of coupled linear integral equations of the Fredholm type for the distribution functions $\rho_1(k)$ and $\rho_2(k)$ for the first and second chains

$$\rho_1(k)+\int_{-Q_2}^{Q_2}dk'\mathcal{R}_1(k,k')\rho_2(k')=\frac{1}{2\pi},$$

$$\rho_2(k)+\int_{-Q_1}^{Q_1}dk'\mathcal{R}_2(k,k')\rho_1(k')=\frac{1}{2\pi}\tag{11}$$

with the kernels $\mathcal{R}_1(k,k')=\mathcal{R}(k,k';t_1,t_2)$, $\mathcal{R}_2(k,k')=\mathcal{R}(k,k';t_2,t_1)$, and

$$\mathcal{R}(k,k';t_1,t_2)=\frac{1}{2\pi}\frac{t_2^2-t_1^2+J^2+2Jt_2\cos k'}{t_1^2+t_2^2+J^2+2t_1t_2\cos(k+k')+2J(t_1\cos k+t_2\cos k')}.$$

Note, that the kernels of integral equations for rapidities are not reduced to difference ones at $-1+t<J<1-t$, as it takes place in the integrable models,[6] such at $t=0$ (or $t_2=0$) the kernels are simplified $\mathcal{R}_1(k,k')=\frac{1}{2\pi}\frac{J^2-1}{1+J^2+2J\cos k}$ and $\mathcal{R}_2(k,k')=\frac{1}{2\pi}$.

Numerically solving integral Eq. (11), we calculate the ground state of the chains for different filling, the parameters of the interactions: $n_{1,2}=N_{1,2}/L=\int_{-Q_{1,2}}^{Q_{1,2}}dk\rho_{1,2}(k)$ are the densities of fermions in the chains,

$$\varepsilon_{1,2}=-2t_{1,2}\int_{-Q_{1,2}}^{Q_{1,2}}dk\cos(k)\rho_{1,2}(k)-w_{1,2}n_{1,2}$$

are the densities of the ground-state energy of the chains, $\varepsilon=\varepsilon_1+\varepsilon_2$ and $n=n_1+n_2$ are the density of the total ground-state energy and the total density of fermions. We consider the subbands with coincident band centers or the case $w_1=w_2=J$.

Another way to describe the system is in the terms of integral equations for excitation energies $\epsilon_1(k)$ and $\epsilon_2(k)$,

$$\epsilon_1(k)+\int_{-Q_2}^{Q_2}dk'\mathcal{R}_2(k',k)\epsilon_2(k')=-2t_1\cos k-\mu,$$

$$\epsilon_2(k)+\int_{-Q_1}^{Q_1}dk'\mathcal{R}_1(k',k)\epsilon_1(k')=-2t_2\cos k-\mu,\tag{12}$$

where μ is the chemical potential.

(1) The definitions of "intermediate metallic state" and "interchain ferromagnetic state" in Fig. 1 are not clear. It should be explained what "ferromagnetism" means in the context of the spinless fermion model. If ferromagnetism means the state in which one of two chains is fully occupied by fermions, and other is empty, the author should plainly explain so.

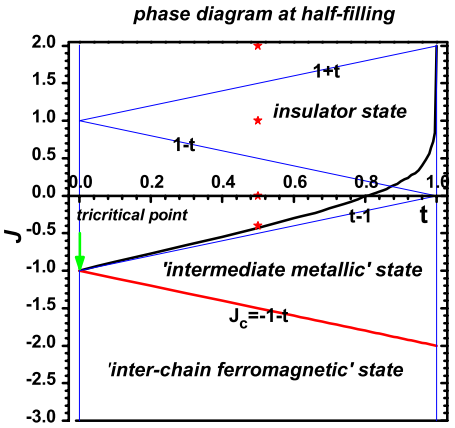

FIG. 1. (Color online) The ground-state phase diagram at half filling in the coordinates t and J, the regions of the parameters corresponding to the rapidity parametrization of the Bethe equations are lined out.

The distribution functions $\rho_{1,2}(k)$ are positive definite functions for all momenta $k^{(1)}$ in the interval $[-Q_1,Q_1]$ and $k^{(2)}$ in the interval $[-Q_2,Q_2]$. The ground state of the coupled chains is defined by values of the coupling constants and filling. Let us consider the limit cases for the parameters of the model Hamiltonian at half filling, namely, $t=0,1$ and $J\to\pm\infty$. The limit $t=1$ corresponds to metal state (at $-2\le J\le 2$) with half-filled chains of spinless fermions, another limit $t=0$ at $J=0$ corresponds to insulator state, whereas the hopping along of the chain with bare band width equaled 2 is forbidden via equaled zero correlated hopping integral and fermions of another chain (with bare band width equaled t) are frozen. Other limits $J\to\infty$ and $J\to-\infty$ correspond to antiferromagnetic gaped state and interchain ferromagnetic state, respectively. Below we calculate values of the parameters t and J that correspond to the metal-insulator phase transition at half filling. Half filling, i.e., $n=1$ corresponds to $Q_1=Q_2=\pi/2$ at $t=1$, $J=0$, $\pi/2<Q_2<\pi$ (partially filled subband) in metallic state, and $Q_2=\pi$ (fully filled subband) in insulator state. The ground-state phase diagram of model at half filling is shown in Fig. 1. The structure of the phase diagram is similar to the same of HC, where an insulator state at $\Delta>2$ and a ferromagnetic state at $\Delta<-2$ are separated by a metallic (antiferromagnetic gapless) state at $-2\le\Delta\le 2$.[7] The ferromagnetic state of HC is described by the string solution (9), the metallic and insulator states are defined by real solutions for the momentum with $|k|<\pi$ and $|k|\le\pi$, respectively. According to Fig. 1 between critical values of $J=\pm 2$ these is a region [in the (t,J) plane] of metallic state (compare with the antiferromagnetic gapless state at $-2\le\Delta\le 2$ in HC) that we shall call as "intermediate metallic" state. A straight line $J_c=-1-t$ follows from the stability of the real solutions of integral Eqs. (11) and (12) for the momenta k_j. The phase state below this line is defined by the complex string solutions of fermions of different chains, Eq. (9), similar to HC at $\Delta<-2$ for one chain. We shall call this state as interchain ferromagnetic phase state. An upper curve ($J=2$ at $t=1$, $J=0$ at $t=0.8$, and $J=-1$ at t

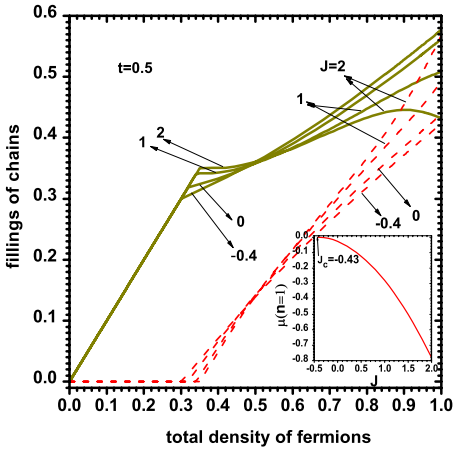

FIG. 2. (Color online) The fillings of the chains as a function of the total density of fermions at $t=0.5$ and different values of $J=2;1;0;-0.4$. Solid and dashed curves correspond to the fillings of the chains with bare bandwidths 2 and $2t$. The chemical potential $\mu(n=1)$ at half filling and $t=0.5$ as a function of J, the point $J_0=-0.43$ corresponds to insulator "intermediate metal" phase transition in Fig. 1.

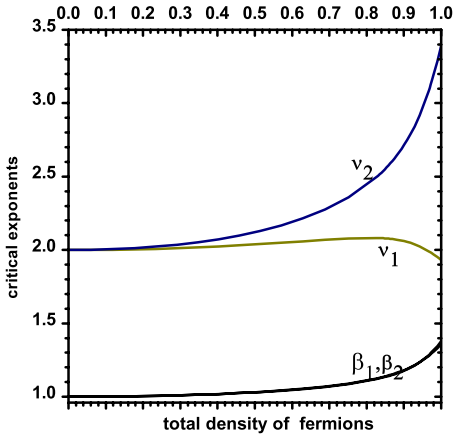

FIG. 3. (Color online) Critical exponents ν_1, ν_2 of the density-density correlation function and β_1, β_2 of the one-particle correlation function as a function of the total density of fermions at $t=0.5$ and $J=1$.

$=0$) separates gapped and gapless states of the model (here the $t=1$ limit coincides with $\Delta=2$ in HC). Two modes of excitations $\epsilon_1(k)$ and $\epsilon_2(k)$ are gapless in the metallic state, and $\epsilon_2(k)$ mode has a gap in the insulator state.

The coexistence of a tricritical point is the peculiarity of the metal-insulator phase transition in the model considered. The point with coordinates $t=0$, $J=-1$ is the tricritical point (see Fig. 1), the kernels of Eqs. (11) and (12) are simplified in the tricritical point, namely, $\mathcal{R}_1(k,k')=-\delta(k)$ and $\mathcal{R}_2(k,k')=\frac{1}{2\pi}$. The solutions of Eq. (11) obtained in the tricritical point from the insulator state at $t=0$ and $J\to -1+0$ (see the direction of the arrow in Fig. 1) have the following form $\rho_1(k)=\frac{1}{2\pi}+\frac{1}{3}\delta(k)$, $\rho_2(k)=\frac{1}{6\pi}$, $Q_1=\frac{\pi}{3}$, $Q_2=\pi$, and $n_1=\frac{2}{3}$, $n_2=\frac{1}{3}$. In the tricritical point equations for excitation energies $\epsilon_1(k)$ and $\epsilon_2(k)$ are reduced to the following ones:

$$\epsilon_1(k)+\frac{1}{2\pi}\int_{-Q_2}^{Q_2}dk'\epsilon_2(k')=-2\cos k-\mu+1,$$

$$\epsilon_2(k)-\int_{-Q_1}^{Q_1}dk'\delta(k')\epsilon_1(k')=-\mu+1. \tag{13}$$

According to Eq. (13), $\epsilon_2(k)$ mode is dispersionless and $\epsilon_1(k)=-2\cos k+1$.

The model undergoes a metal-insulator phase transition at partial fillings of the chains (see Fig. 2). Due to strong interaction the fermion subband is not associated with the chain, therefore the filling of the chain is not correlated with the band filling. The fillings of the chains are calculated for the parameters of the interactions denoted in Fig. 1 by the stars. According to the calculations the difference in the occupations of the chains increases at $J\to J_c$.

In the metallic state the behavior of spinless fermions is characterized by two branches of the charge gapless excitations. The critical behavior of the coupled chains is described by a $c=1$ conformal field theory. The correction of the ground-state energy is defined by the Fermi velocities v_1 and v_2 of the charge excitations in the chains $E_0-L\varepsilon=-\frac{\pi}{6L}(v_1+v_2)$.[8] Similarly the energies and momenta of low-lying excitations are given by $E-L\varepsilon=\frac{2\pi}{L}[v_1(\Delta_1^+ +\Delta_1^-)+v_2(\Delta_2^+ +\Delta_2^-)]$, $\mathcal{P}-\mathcal{P}_0=2k_F^{(1)}D_1+2k_F^{(2)}D_2+\frac{2\pi}{L}(\Delta_1^+-\Delta_1^-+\Delta_2^+-\Delta_2^-)$ with the conformal dimensions $2\Delta_l^{\pm}=(\chi_{ll}D_l+\chi_{l'l}D_{l'}\pm\frac{\chi_{l'l'}\Delta N_l-\chi_{ll'}\Delta N_{l'}}{2\det\chi})^2+2N_l^{\pm}$ and the Fermi momenta $k_F^{(1,2)}$, here $D_{1,2}=\frac{\Delta N_{1,2}}{2}(\mathrm{mod}1)$ are integers or half-odd integers depending on the parities of $\Delta N_{1,2}$. The components of the dressed charge $\chi_{ll'}$ are defined via the dressed charge matrix $\chi_{1l}=\chi_{1l}(Q_l)$, $\chi_{2l}=\chi_{2l}(Q_l)$. The dressed charge matrix $\chi_{ll'}(k)$ is calculated using the following set of the integral equations:

$$\chi_{ll}(k)+\int_{-Q_{l'}}^{Q_{l'}}dk'\mathcal{R}_{l'}(k',k)\chi_{ll'}(k')=1,$$

$$\chi_{ll'}(k)+\int_{-Q_l}^{Q_l}dk'\mathcal{R}_l(k',k)\chi_{ll}(k')=0. \tag{14}$$

Numerically solving integral Eqs. (11) and (14) we calculate the components of the dressed charge matrix as functions of the density of spinless fermions for different values of the parameters of the interactions. The critical exponents describing the long-distance asymptotics of the equal-time correlation functions are determined by the conformal dimensions $2(\Delta_1^+ +\Delta_1^-)+2(\Delta_2^+ +\Delta_2^-)$. The leading asymptotics of the density-density correlation function in the chains $\langle n(x)n(0)\rangle \simeq n^2+A_0x^{-2}+A_1^{(l)}x^{-\nu_l}\cos(2k_F^{(l)}x)$ are defined by intrachain critical exponents $\nu_l=2(\chi_{ll}^2+\chi_{ll'}^2)$. The long-distance asymptotics of the one-particle correlation function are defined by the critical exponents $\beta_{1,2}$ for each chain $\langle c^{\dagger}(x)c(0)\rangle \simeq B_lx^{-\beta_l}\cos(k_F^{(l)}x)$, $\beta_l=\frac{1}{2}(\chi_{ll}^2+\chi_{ll'}^2)+\frac{1}{2}\frac{\chi_{ll}^2+\chi_{ll'}^2}{(\chi_{ll}\chi_{l'l'}-\chi_{ll'}\chi_{l'l})^2}$. According to simulations the values of β_1 and β_2 are similar functions of the total density of fermions, the maximum value is reached at $n\to 1$ (see Fig. 3). In the tricritical point the second branch of excitations is dispersionless with $v_2=0$, as

result, a phase state of fermions is characterized by one gapless mode. The critical behavior of the correlation functions is determined by a dressed charge $\chi_1(k)=\chi_1=\frac{1}{2}$, which is the solution of the following equation: $\chi_1(k)+\int_{-Q_1}^{Q_1}dk'\,\delta(k')\chi_1(k')=1$. The asymptotic behavior of the density-density correlation function is described by the critical exponent $\nu_1=2\chi_1^2=\frac{1}{2}$. The leading asymptotic of the one-particle correlation function is defined by an abnormal large value of the critical exponent $\beta_1=2.125$. Such large value of β_1 and small value of ν_1 is realized in the model of spinless fermions with a hard-core radius equaled to the lattice constant at half filling.[9]

IV. CONCLUSIONS

We have discussed in depth the model of two coupled nondegenerate free fermion chains interacting via the correlated hopping and interchain Coulomb interaction. The model has a rich ground-state phase diagram at half filling: the intermediate metallic state separates the insulator and interchain ferromagnetic phase states, the curves of the phase transitions are defined by the parameters of the interactions. The phase diagram is characterized by the tricritical point, where these three phase states coexist. In the tricritical point a Luttinger liquid state is characterized by abnormal large density-density correlations and correlations of the field operator.

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